SHORTER COMMUNICATIONS

A NOTE ON THE ENHANCEMENT OF THE DIFFUSION LIMITED VAPORIZATION RATES BY CONDENSATION WITHIN THE THERMAL BOUNDARY LAYER

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NOMENCLATURE

B, C,	constants in the vapor pressure/temperature relationship, expressed as $m = \exp(B - C/T)$;
C^*,	mole fraction;
C_p,	specific heat;
D,	diffusion coefficient;
d,	droplet diameter, or characteristic linear dimension of flow system;
g,	acceleration due to gravity;
h,	heat-transfer coefficient;
H,	heat of condensation, or evaporation;
h_D,	mass-transfer coefficient;
j'', j_{max},	defined in [1];
k,	thermal conductivity;
m,	mass fraction;
$\dot{m}''$,	evaporation rate per unit area of surface;
$\dot{m}'''_c$,	condensation rate per unit volume in boundary layer;
M,	molecular weight;
p,	vapor pressure;
$\dot{q}'''$,	rate of heat liberation per unit volume in boundary layer;
R,	gas constant;
T,	temperature;
$\mathbf{v}$,	local velocity vector in boundary layer;
Nu,	Nusselt number, $= hd/k$;
Sh,	Sherwood number, $= h_D d/\rho D$;
Le,	Lewis number, $= \varkappa/D$;
Gr,	Grashof number, $= \beta \Delta\theta g d^3/\nu^2$;
Pr,	Prandtl number, $= C_p \mu/k$.

Greek symbols

β,	coefficient of thermal expansion;
μ,	viscosity;
ν,	kinematic viscosity;
ρ,	density;
ξ,	modified temperature function, $= (H/C_p)m + T$;
$\varkappa$,	thermal diffusivity.

Subscripts

f,	film;
s,	values in the free stream;
w,	values at the vaporizing surface;
ef,	effective value in boundary layer;
Ni,	nickel;
Fe,	iron;
He,	helium.

IN A RECENT paper Rosner [1] presented a formulation for describing the effect of condensation on diffusion limited vaporization. This paper provides a rather more rigorous representation of phenomena, first reported by Turkdogan [2, 3] and Toop [4], in the regime where the mass fraction of the vaporizing species is small.

The theoretical predictions in Rosner's paper were compared with Turkdogan's experimental data and quite reasonable qualitative agreement has been obtained.

The purpose of this communication is to draw attention to a previous publication [5] by the authors, which may be more appropriate to situations where the vaporizing species is present in moderate concentrations and thus may be regarded as one extending the domain of validity of Rosner's treatment.

These considerations may be particularly relevant, as a good portion of the data reported by Turkdogan do indeed correspond to moderate to high concentrations of the diffusing species.

FORMULATION

In their previous treatment, the authors considered the

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differential heat and mass balance equations in the flowing gas in the vicinity of an evaporating body:

$$\nabla \cdot \{ \varkappa \nabla T \} - \mathbf{v} \cdot \nabla T = -\frac{\dot{q}_c'''}{c_p \rho} \qquad (1)$$

$$\nabla \cdot (D \nabla m) - \mathbf{v} \nabla m = \frac{\dot{m}_c'''}{\rho} \qquad (2)$$

where:

$$\dot{q}_c''' = H \dot{m}_c'''. \qquad (3)$$

For situations in which c_p is constant, and the Lewis number is equal to unity ($D = \varkappa$), equation (2) can be multiplied by H/c_p and added to equation (1) to give:

$$\nabla \cdot (\varkappa \nabla \zeta) - \mathbf{v} \cdot \nabla \zeta = 0 \qquad (4)$$

where

$$\zeta = \frac{H}{c_p} m + T. \qquad (5)$$

Thus we can say:

$$\frac{[\nabla \zeta]_w}{\zeta_w - \zeta_s} = \left[\frac{[\nabla T]_w}{T_w - T_s} \right]_{\dot{m}''=0} = -\frac{Nu}{d} \qquad (6)$$

where d is a characteristic linear dimension of the flow system. Provided that we know a relation between m and T:

$$m = f_1(T) \qquad (7)$$

we can obtain

$$\nabla \zeta = \frac{H}{c_p} \nabla m + \nabla T = \left\{ \frac{H}{c_p} + \frac{1}{f_1'(T)} \right\} \nabla m \qquad (8)$$

which can be substituted into equation (6) to give:

$$[\nabla m]_w = -\frac{Nu}{d} \left\{ \frac{\zeta_w - \zeta_s}{\dfrac{H}{c_p} + \dfrac{1}{[f_1'(T)]_w}} \right\}. \qquad (9)$$

But

$$[\nabla m]_w = -\frac{Sh}{d} (m_w - m_s) \qquad (10)$$

whence:

$$\frac{Sh}{Nu} = \frac{\zeta_w - \zeta_s}{\dfrac{H}{c_p} + \dfrac{1}{[f_1'(T)]_w}} \left\{ \frac{1}{m_w - m_s} \right\}. \qquad (11)$$

Equations (6, 9, 10, 11) can be considered to apply to local or to mean values.

If we can assume that the vapor species is in equilibrium with its condensate throughout the boundary layer, equation (7) is the standard vapour pressure relation:

$$m = \exp \left(B - \frac{C}{T_w} \right) \qquad (12)$$

so that

$$[f_1'(T)]_w = \frac{C}{T_w^2} \exp \left(B - \frac{C}{T_w} \right) = \frac{C}{T_w^2} m_w. \qquad (13)$$

Thus, for the standard situation in which $m_s = 0$, equation (11) becomes:

$$\frac{Sh}{Nu} = \frac{\dfrac{C}{T_w^2} (T_w - T_s) + \dfrac{H}{c_p} m_w}{1 + \dfrac{H}{c_p} m_w \dfrac{C}{T_w^2}}. \qquad (14)$$

Here the term Sh/Nu clearly corresponds to j_{max}''/j_{min}'' defined in Rosner's work.

It is readily shown, that when m_w is small, equation (14) will reduce to:

$$\frac{Sh}{Nu} = \frac{C}{T_w} \left(1 - \frac{T_s}{T_w} \right) \qquad (15)$$

which is an expression, identical to Rosner's equation (17).

The principal difference between equations (14) and (15) is that, in the former, *allowance has been made for the heat given up by the condensing vapor, which for moderate to large mass fractions will reduce the effective enhancement in vaporization brought about by the condensation mechanism.*

This is illustrated in Fig. 1 where Sh is plotted against T_w. Here the discrete points represent Turkdogan's results, the solid line represents the predictions based on equation (14) in the text whereas the broken line corresponds to the interpretation put forward by Rosner in apparent agreement with the results of Turkdogan's earlier work. (the values for this broken line were computed from Fig. 4 in [1]). Finally, the "dash-dotted line" appearing on the top of the graph represents the value of the Sherwood number as calculated from equation (15), which is, as previously mentioned identical to j_{max}'' defined in [1].

It is seen that while the solid line does not provide perfect agreement with the experimental points, it appears to reproduce the overall trend rather more accurately.

It is suggested that this form of representation, effectively in terms of mass transfer coefficients on a linear scale, may be preferable to that adopted by Rosner in Fig. 4 of his publication, where the mass flux was plotted against the surface temperature. In this latter case the exponential dependence of vapor pressure (hence driving force) on temperature is so overwhelming, that the net effects due to condensation are rather more difficult to distinguish.

The details of the calculations leading to the construction of the curves and representation of the experimental points are given in the Appendix.
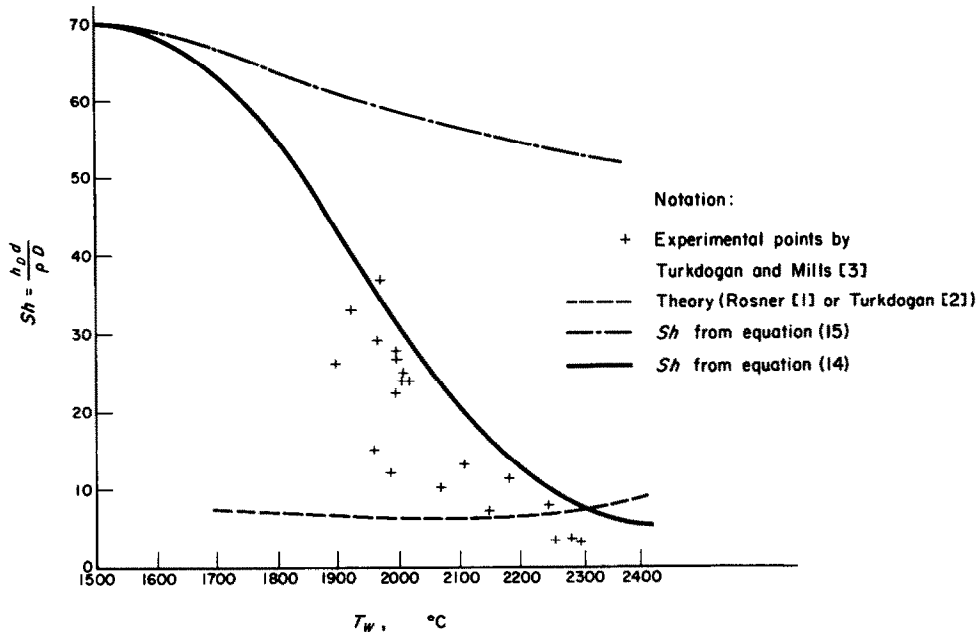
FIG. 1. A Plot of Sh vs. T_w showing the experimental results and also a comparison of the theoretical predictions.

CONCLUSIONS

It is suggested, that while the treatment described in [1–3] is satisfactory for low and very low concentrations of the diffusing species, the method previously described by the authors [5], would be more appropriate at intermediate concentration levels of the vaporizing material; thus the two techniques are complementary, rather than contradictory.

At very high concentration levels even this latter treatment would break down, as bulk flow due to diffusion would also have to be taken into account.

APPENDIX

Calculation of the Values Shown in Fig. 1

1. *Calculation of the theoretical curve*

The experimental points in Fig. 1 were obtained from data by Turkdogan and Mills [3] in a study of the vaporization of Fe/Ni alloys into stagnant helium. As both the vapor pressures and molecular weights of iron and nickel are quite similar, a good approximation for the theoretical prediction may be made by considering the vaporization of pure nickel.

In the absence of externally imposed flow, the Nusselt number, used in calculating the Sherwood number in equation (14), is obtained from the appropriate expression for natural convection:

$$Nu = 2 + 0{\cdot}60\,(Gr)^{\frac{1}{4}}\,(Pr)^{\frac{1}{3}} \qquad (16)$$

In evaluating the property values for substitution into equation (16) the following assumptions were made:

(i) The gas was assigned an effective molecular weight, M_{ef} corresponding to the composition of the helium-nickel mixture at the evaporating surface, i.e.

$$M_{ef} = 4[1 - (C_{Ni}^*)_w] + 59(C_{Ni}^*)_w \qquad (17)$$

(ii) Assuming ideal gas law, the effective heat capacity and density were given as follows:

$$[C_p]_{ef} = \frac{5}{M_{ef}} \qquad (18)$$

and

$$\rho_{ef} = \frac{M_{ef}P_T}{RT_f} \qquad (19)$$

where T_f, the film temperature was taken as the arithmetic mean of the wall and free stream temperatures.

(iii) The equilibrium vapor pressure of nickel was calculated from an equation given by Kubaschewski and Evans [8]:

H

$$\ln (p_{Ni}) = 18\cdot34 - \frac{51\,500}{T} - 0\cdot364 \ln T \qquad (20)$$

(iv) The latent heat of vaporization was taken as: $1\cdot7 \times 10^3$ cal g^{-1}.

Noting the definition of Gr, substitution of these quantities into equation (16) gave Nusselt numbers of 2·9 and 3·1, corresponding to surface temperatures of 1500 and 2400°C respectively· In view of this small variation of Nu with temperature in this instance, a mean value of $\overline{Nu} = 3.0$ was used throughout in the calculations.

Finally, it is readily seen by the comparison of equations (12) and (20) that the value of the constant C is given approximately at 51 500 deg K^{-1}.

2 *Calculation of practical values of the Sherwood number*

Practical values of the Sherwood number shown in Fig. 1 have been calculated from the equation:

$$Sh = \frac{h_D d}{\rho_{ef} D}. \qquad (21)$$

Since both iron and nickel evaporate from the surface of the metal drops, the mass-transfer coefficient has been calculated from the equation:

$$h_D = \frac{\dot{m}''}{(m_{Ni})_w + (m_{Fe})_w}. \qquad (22)$$

Where $\dot{m}''$ is the practically measured total evaporation rate, and $(m_{Ni})_w$ and $(m_{Fe})_w$ are the equilibrium mass fractions of nickel and iron in the gas phase in contact with the metal surface. These values have been calculated from

the equilibrium data presented by Kubaschewski and Evans [8], and the activity coefficient data presented Hultgren *et al.* [9]. The values of $d/\rho_{ef}D$ has been calculated from the density values determined from equation (19) and diffusion coefficient data presented by Turkdogan [2].

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THERMAL TURBULENCE CHARACTERISTICS IN SODIUM–POTASSIUM

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NOMENCLATURE

$\overline{T}$, mean temperature [°F];

T_w, extrapolated wall temperature [°F];

T_c, centre line temperature [°F];

T_m, bulk mixing cup temperature [°F];

y, distance measured from the wall [ft];

R_0, radius of the pipe [ft];

$\sqrt{(\overline{\theta'^2})}$, RMS value of temperature fluctuation [°F];

$\sqrt{(\theta'^2_{max})}$, maximum value of RMS temperature fluctua-

tion for one set of data;

U_b, bulk average velocity [ft/s];

$\sqrt{(\overline{u'^2})}$, RMS value of velocity fluctuation [ft/h];

$\sqrt{(u'^2_{max})}$, maximum value of velocity fluctuation for one set of data [ft/h];

$(q/A)_w$, wall heat flux [Btu·ft^2];

k, thermal conductivity [Btu/h·ft·°F];

U^*, friction velocity [ft/h];

ν, kinematic viscosity [ft^2/h];